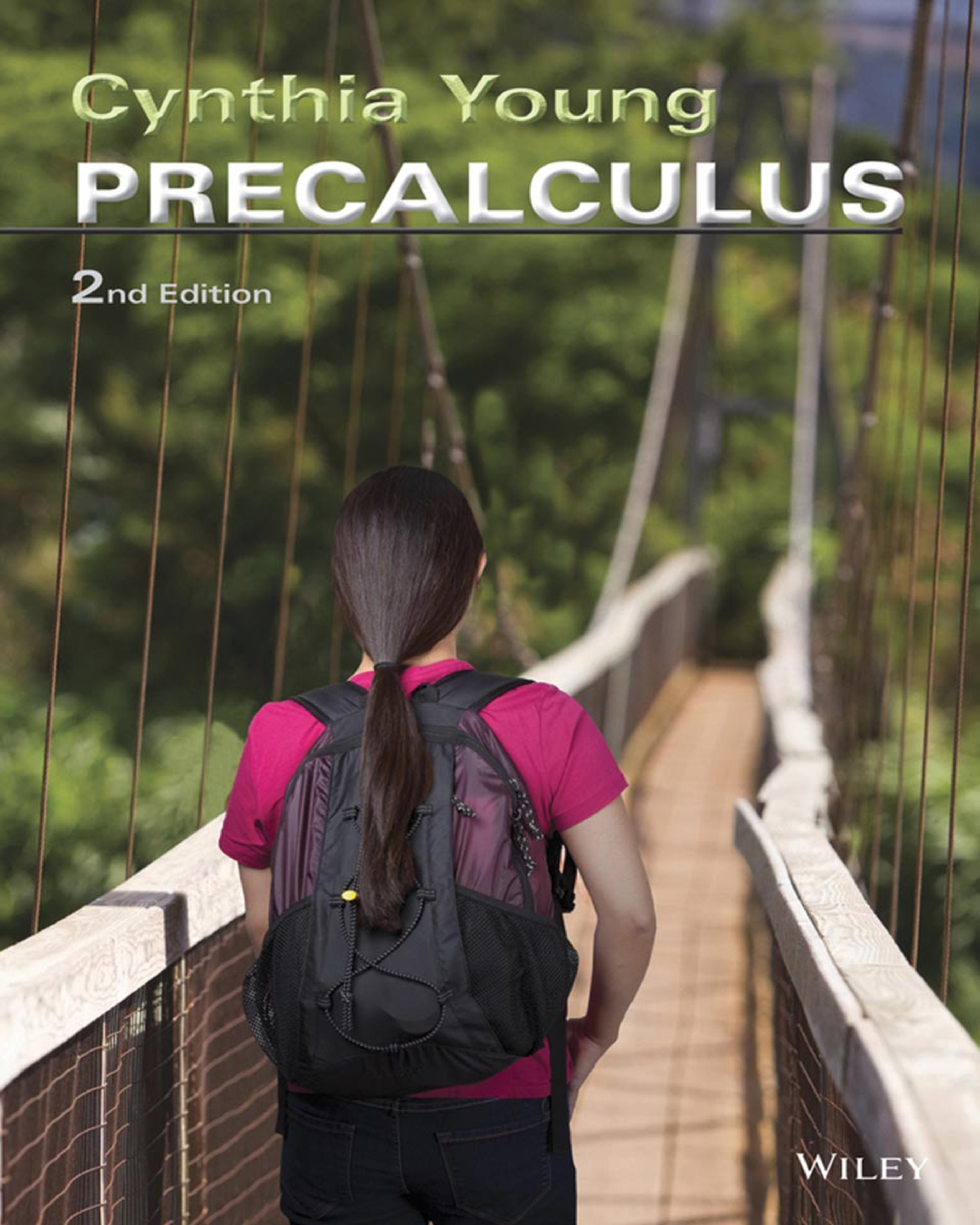

Cynthia Young

PRECALCULUS

2nd Edition

WILEY

Precalculus

Second Edition

CYNTHIA Y. YOUNG | *Professor of Mathematics*
UNIVERSITY OF CENTRAL FLORIDA

WILEY

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For Christopher and Caroline

About the Author

Cynthia Y. Young is a native of Tampa, Florida. She currently is a Professor of Mathematics at the University of Central Florida (UCF) and the author of *College Algebra*, *Trigonometry*, *Algebra and Trigonometry*, and *Precalculus*. She holds a B.A. degree in Secondary Mathematics Education from the University of North Carolina (Chapel Hill), an M.S. degree in Mathematical Sciences from UCF, and both an M.S. in Electrical Engineering and a Ph.D in Applied Mathematics from the University of Washington. She has taught high school in North Carolina and Florida, developmental mathematics at Shoreline Community College in Washington, and undergraduate and graduate students at UCF. Dr. Young's two main research interests are laser propagation through random media and improving student learning in STEM. She has authored or co-authored over 60 books and articles and been involved in over $2.5M in external funding. Her atmospheric propagation research was recognized by the Office of Naval Research Young Investigator award, and in 2007 she was selected as a Fellow of the International Society for Optical Engineers. She is currently the co-director of UCF's EXCEL program whose goal is to improve the retention of STEM majors.

Although Dr. Young excels in research, she considers teaching her true calling. She has been the recipient of the UCF Excellence in Undergraduate Teaching Award, the UCF Scholarship of Teaching and Learning Award, and a two-time recipient of the UCF Teaching Incentive Program. Dr. Young is committed to improving student learning in mathematics and has shared her techniques and experiences with colleagues around the country through talks at colleges, universities, and conferences.

Dr. Young and her husband, Dr. Christopher Parkinson, enjoy spending time outdoors and competing in Field Trials with their Labrador Retrievers. *Laird's Cynful Wisdom* (call name "*Wiley*") is titled in Canada and currently pursuing her U.S. title. *Laird's Cynful Ellegance* (call name "*Ellie*") was a finalist in the Canadian National in 2009 and is retired (relaxing at home).

Dr. Young is pictured here with Ellie's 2011 litter of puppies!

Bonnie Farris

Preface

As a mathematics professor I would hear my students say, "I understand you in class, but when I get home I am lost." When I would probe further, students would continue with "I can't read the book." As a mathematician I always found mathematics textbooks quite easy to read—and then it dawned on me: don't look at this book through a mathematician's eyes; look at it through the eyes of students who might not view mathematics the same way that I do. What I found was that the books were not at all like my class. Students understood me in class, but when they got home they couldn't understand the book. It was then that the folks at Wiley lured me into writing. My goal was to write a book that is seamless with how we teach and is an ally (not an adversary) to student learning. I wanted to give students a book they could read without sacrificing the rigor needed for conceptual understanding. The following quote comes from a reviewer of this third edition when asked about the rigor of the book:

> *I would say that this text comes across as a little less rigorous than other texts, but I think that stems from how easy it is to read and how clear the author is. When one actually looks closely at the material, the level of rigor is high.*

Distinguishing Features

Four key features distinguish this book from others, and they came directly from my classroom.

Parallel Words and Math

Have you ever looked at your students' notes? I found that my students were only scribbling down the mathematics that I would write—never the words that I would say in class. I started passing out handouts that had two columns: one column for math and one column for words. Each Example would have one or the other; either the words were there and students had to fill in the math, or the math was there and students had to fill in the words. If you look at the Examples in this book, you will see that the words (your voice) are on the left and the mathematics is on the right. In most math books, when the author illustrates an Example, the mathematics is usually down the center of the page, and if the students don't know what mathematical operation was performed, they will look to the right for some brief statement of help. That's not how we teach; we don't write out an Example on the board and then say, "Class, guess what I just did!" Instead we lead our students, telling them what step is coming and then performing that mathematical step *together*—and reading naturally from left to right. Student reviewers have said that the Examples in this book are easy to read; that's because *your* voice is right there with them, working through problems *together*.

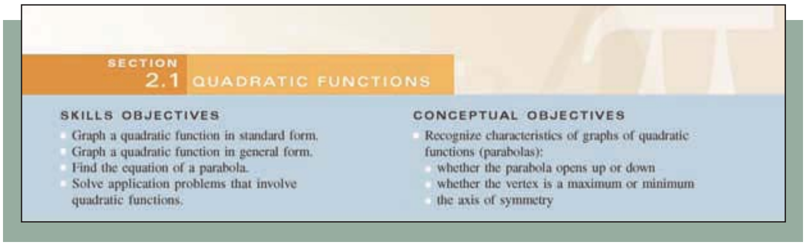
EXAMPLE 1 Graphing a Quadratic Function Given in Standard Form

Graph the quadratic function $f(x) = (x - 3)^2 - 1$.

Solution:

STEP 1 The parabola opens up.	$a = 1$, so $a > 0$
STEP 2 Determine the vertex.	$(h, k) = (3, -1)$
STEP 3 Find the y-intercept.	$f(0) = (-3)^2 - 1 = 8$ $(0, 8)$ corresponds to the y-intercept

Skills and Concepts (Learning Objectives and Exercises)



SECTION 2.1 QUADRATIC FUNCTIONS

SKILLS OBJECTIVES

- Graph a quadratic function in standard form.
- Graph a quadratic function in general form.
- Find the equation of a parabola.
- Solve application problems that involve quadratic functions.

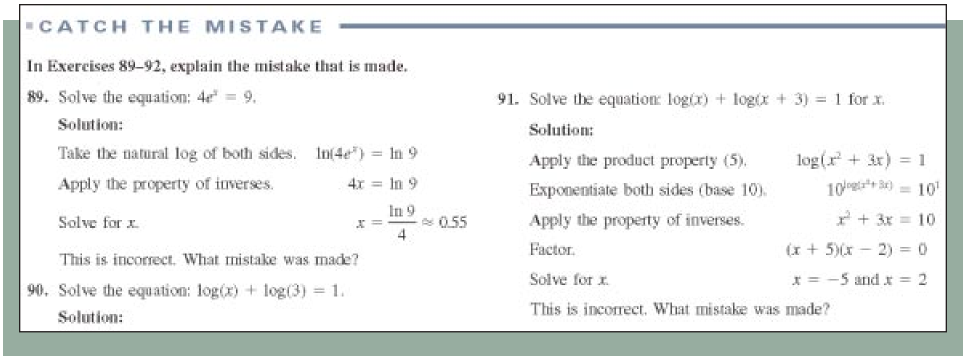
CONCEPTUAL OBJECTIVES

- Recognize characteristics of graphs of quadratic functions (parabolas):
  - whether the parabola opens up or down
  - whether the vertex is a maximum or minimum
  - the axis of symmetry

In my experience as a mathematics teacher/instructor/professor, I find skills to be on the micro level and concepts on the macro level of understanding mathematics. I believe that too often skills are emphasized at the expense of conceptual understanding. I have purposely separated *learning objectives* at the beginning of every section into two categories: *skills objectives*—what students should be able to do; and *conceptual objectives*—what students should understand. At the beginning of every class I discuss the learning objectives for the day—both skills and concepts. These are reinforced with both skills exercises and conceptual exercises.

Catch the Mistake

Have you ever made a mistake (or had a student bring you his or her homework with a mistake) and you go over it and over it and can't find the mistake? It's often easier to simply take out a new sheet of paper and solve it from scratch again than it is to actually find the mistake. Finding the mistake demonstrates a higher level of understanding. I include a few *Catch the Mistake* exercises in each section that demonstrate a common mistake that I have seen in my experience. I use these in class (either as a whole or often in groups), which leads to student discussion and offers an opportunity for formative assessment in real time.



CATCH THE MISTAKE

In Exercises 89–92, explain the mistake that is made.

89. Solve the equation: $4e^x = 9$.

Solution:

Take the natural log of both sides. $\ln(4e^x) = \ln 9$

Apply the property of inverses. $4x = \ln 9$

Solve for x. $x = \frac{\ln 9}{4} \approx 0.55$

This is incorrect. What mistake was made?

90. Solve the equation: $\log(x) + \log(3) = 1$.

Solution:

91. Solve the equation: $\log(x) + \log(x + 3) = 1$ for x.

Solution:

Apply the product property (5). $\log(x^2 + 3x) = 1$

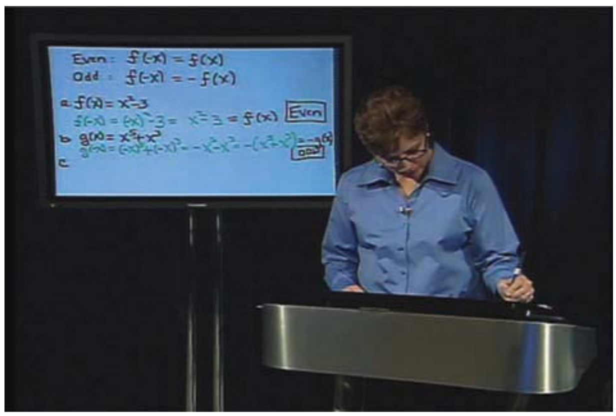
Exponentiate both sides (base 10). $10^{\log(x^2+3x)} = 10^1$

Apply the property of inverses. $x^2 + 3x = 10$

Factor. $(x + 5)(x - 2) = 0$

Solve for x. $x = -5$ and $x = 2$

This is incorrect. What mistake was made?



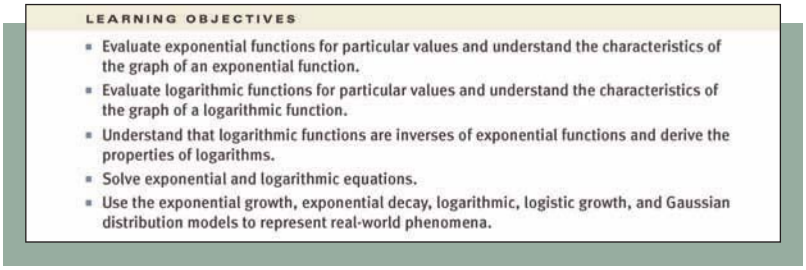
Lecture Videos by the Author

To ensure consistency in the students' learning experiences, I authored the videos myself. Throughout the book wherever a student sees the video icon, that indicates a video. These videos provide a mini lecture in that the chapter openers and chapter summaries are more like class discussion and selected Examples. Your Turns throughout the book also have an accompanying video of me working out that exact problem.

New to the Second Edition

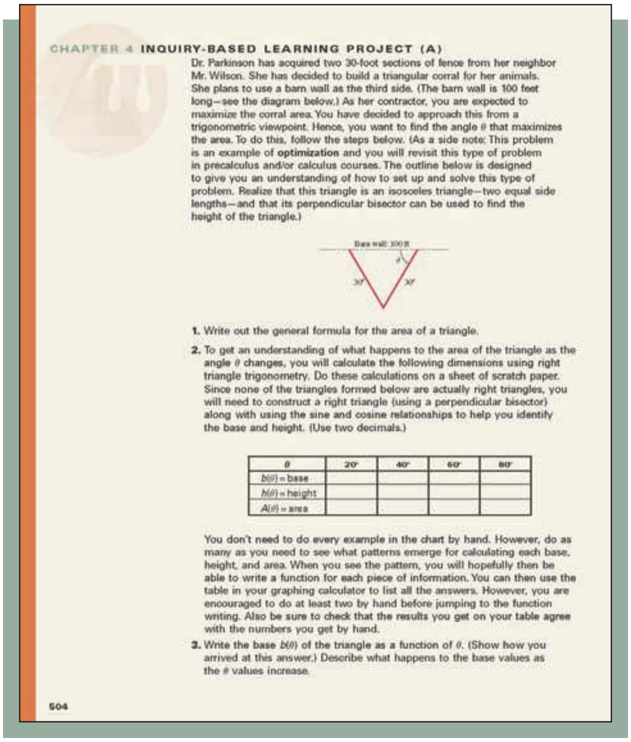
The first edition was *our* book, and this second edition is *our even better* book. I've incorporated some specific line-by-line suggestions from reviewers throughout the exposition, added some new Examples, and added over 200 new Exercises. The three main global upgrades to the second edition are a new Chapter Map with Learning Objectives, End-of-chapter Inquiry-Based Learning Projects, and additional Applications Exercises in areas such as Business, Economics, Life Sciences, Health Sciences, and Medicine. A section (0.8*) on Linear Regression was added, as well as some technology exercises on Quadratic, Exponential, and Logarithmic Regression.

Learning Objectives


LEARNING OBJECTIVES

- Evaluate exponential functions for particular values and understand the characteristics of the graph of an exponential function.
- Evaluate logarithmic functions for particular values and understand the characteristics of the graph of a logarithmic function.
- Understand that logarithmic functions are inverses of exponential functions and derive the properties of logarithms.
- Solve exponential and logarithmic equations.
- Use the exponential growth, exponential decay, logarithmic, logistic growth, and Gaussian distribution models to represent real-world phenomena.

Inquiry-Based Learning Projects


CHAPTER 4 INQUIRY-BASED LEARNING PROJECT (A)

Dr. Parkinson has acquired two 30-foot sections of fence from her neighbor Mr. Wilson. She has decided to build a triangular corral for her animals. She plans to use a barn wall as the third side. (The barn wall is 100 feet long—see the diagram below.) As her contractor, you are expected to maximize the corral area. You have decided to approach this from a trigonometric viewpoint. Hence, you want to find the angle θ that maximizes the area. To do this, follow the steps below. (As a side note: This problem is an example of **optimization** and you will revisit this type of problem in precalculus and/or calculus courses. The outline below is designed to give you an understanding of how to set up and solve this type of problem. Realize that this triangle is an isosceles triangle—two equal side lengths—and that its perpendicular bisector can be used to find the height of the triangle.)

1. Write out the general formula for the area of a triangle.
2. To get an understanding of what happens to the area of the triangle as the angle θ changes, you will calculate the following dimensions using right triangle trigonometry. Do these calculations on a sheet of scratch paper. Since none of the triangles formed below are actually right triangles, you will need to construct a right triangle (using a perpendicular bisector) along with using the sine and cosine relationships to help you identify the base and height. (Use two decimals.)

θ	20°	40°	60°	80°
$b(\theta)$ = base				
$h(\theta)$ = height				
$A(\theta)$ = area				

You don't need to do every example in the chart by hand. However, do as many as you need to see what patterns emerge for calculating each base, height, and area. When you see the pattern, you will hopefully then be able to write a function for each piece of information. You can then use the table in your graphing calculator to list all the answers. However, you are encouraged to do at least two by hand before jumping to the function writing. Also be sure to check that the results you get on your table agree with the numbers you get by hand.

3. Write the base $b(\theta)$ of the triangle as a function of θ. (Show how you arrived at this answer.) Describe what happens to the base values as the θ values increase.

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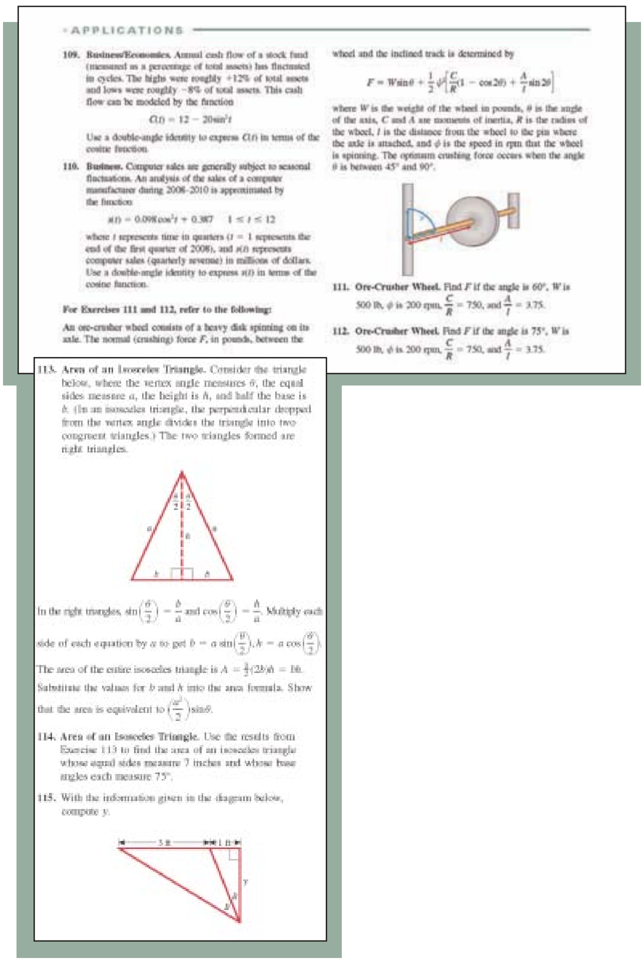
Applications to Business, Economics, Health Sciences, and Medicine


APPLICATIONS

109. **Business/Economics.** Annual cash flow of a stock fund (measured as a percentage of total assets) has fluctuated in cycles. The highs were roughly +12% of total assets and lows were roughly −8% of total assets. This cash flow can be modeled by the function

$$C(t) = 12 - 20\sin^2 t$$

Use a double-angle identity to express $C(t)$ in terms of the cosine function.

110. **Business.** Computer sales are generally subject to seasonal fluctuations. An analysis of the sales of a computer manufacturer during 2008–2010 is approximated by the function

$$s(t) = 0.098\cos^2 t + 0.387 \quad 1 \le t \le 12$$

where t represents time in quarters ($t = 1$ represents the end of the first quarter of 2008), and $s(t)$ represents computer sales (quarterly revenue) in millions of dollars. Use a double-angle identity to express $s(t)$ in terms of the cosine function.

For Exercises 111 and 112, refer to the following:

An ore-crusher wheel consists of a heavy disk spinning on its axle. The normal (crushing) force F, in pounds, between the wheel and the inclined track is determined by

$$F = W\sin\theta + \frac{1}{2}\phi^2\left[\frac{C}{R}(1 - \cos 2\theta) + \frac{A}{l}\sin 2\theta\right]$$

where W is the weight of the wheel in pounds, θ is the angle of the axis, C and A are moments of inertia, R is the radius of the wheel, l is the distance from the wheel to the pin where the axle is attached, and ϕ is the speed in rpm that the wheel is spinning. The optimum crushing force occurs when the angle θ is between 45° and 90°.

111. **Ore-Crusher Wheel.** Find F if the angle is 60°, W is 500 lb, ϕ is 200 rpm, $\frac{C}{R} = 750$, and $\frac{A}{l} = 3.75$.

112. **Ore-Crusher Wheel.** Find F if the angle is 75°, W is 500 lb, ϕ is 200 rpm, $\frac{C}{R} = 750$, and $\frac{A}{l} = 3.75$.

113. **Area of an Isosceles Triangle.** Consider the triangle below, where the vertex angle measures θ, the equal sides measure a, the height is h, and half the base is b. (In an isosceles triangle, the perpendicular dropped from the vertex angle divides the triangle into two congruent triangles.) The two triangles formed are right triangles.

In the right triangles, $\sin\left(\frac{\theta}{2}\right) = \frac{b}{a}$ and $\cos\left(\frac{\theta}{2}\right) = \frac{h}{a}$. Multiply each side of each equation by a to get $b = a\sin\left(\frac{\theta}{2}\right)$, $h = a\cos\left(\frac{\theta}{2}\right)$. The area of the entire isosceles triangle is $A = \frac{1}{2}(2b)h = bh$. Substitute the values for b and h into the area formula. Show that the area is equivalent to $\left(\frac{a^2}{2}\right)\sin\theta$.

114. **Area of an Isosceles Triangle.** Use the results from Exercise 113 to find the area of an isosceles triangle whose equal sides measure 7 inches and whose base angles each measure 75°.

115. With the information given in the diagram below, compute y.

FEATURE	BENEFIT TO STUDENT
Chapter Opening Vignette	Piques the student's interest with a real-world application of material presented in the chapter. Later in the chapter, the same concept from the vignette is reinforced.
Chapter Overview, Flowchart, and Learning Objectives	Students see the big picture of how topics relate and overarching learning objectives are presented.
Skills and Conceptual Objectives	Skills objectives represent what students should be able to do. Conceptual objectives emphasize a higher level global perspective of concepts.
Clear, Concise, and Inviting Writing Style, Tone, and Layout	Students are able to *read* this book, which reduces math anxiety and promotes student success.
Parallel Words and Math	Increases students' ability to read and understand examples with a seamless representation of their instructor's class (instructor's voice and what they would write on the board).
Common Mistakes	Addresses a different learning style: teaching by counter-example. Demonstrates common mistakes so that students understand why a step is incorrect and reinforces the correct mathematics.
Color for Pedagogical Reasons	Particularly helpful for visual learners when they see a function written in red and then its corresponding graph in red or a function written in blue and then its corresponding graph in blue.
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Supplements

Instructor Supplements

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- Contains worked out solutions to all exercises in the text.

Instructor's Manual

Authored by Cynthia Young, the manual provides practical advice on teaching with the text, including:

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- sample syllabi
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Acknowledgments

I want to express my sincerest gratitude to the entire Wiley team. I've said this before, and I will say it again: Wiley is the right partner for me. There is a reason that my dog is named Wiley—she's smart, competitive, a team player, and most of all, a joy to be around. There are several people within Wiley to whom I feel the need to express my appreciation: first and foremost to Laurie Rosatone who convinced Wiley Higher Ed to invest in a young assistant professor's vision for a series and who has been unwavering in her commitment to student learning. To my editor Joanna Dingle whose judgment I trust in both editorial and preschool decisions; thank you for surpassing my greatest expectations for an editor. To the rest of the ladies on the math editorial team (Jen Brady, Liz Baird, and Courtney Welsh), you are all first class! This revision was planned and executed exceptionally well thanks to you three. To the math marketing manager, Kimberly Kanakes, thank you for helping reps tell my story. To Ken Santor, thank you for your attention to detail. And finally, I'd like to thank all of the Wiley reps: thank you for your commitment to my series and your tremendous efforts to get professors to adopt this book for their students.

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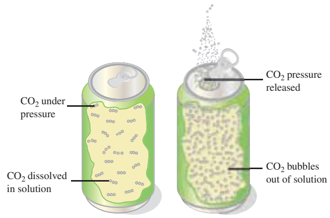

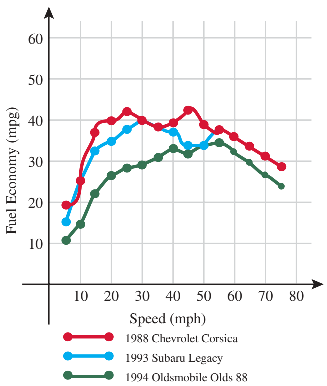


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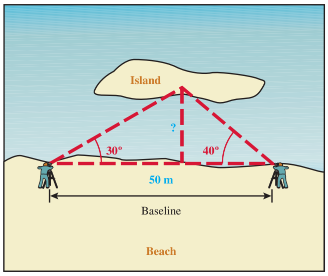

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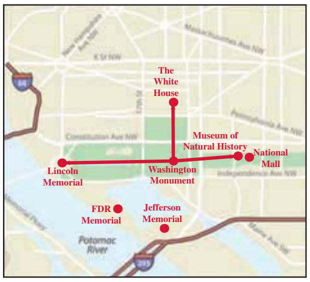
6 Analytic Trigonometry 588



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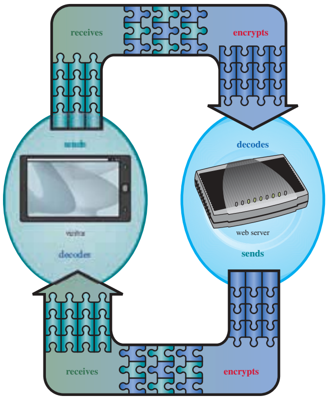
7 Vectors, the Complex Plane, and Polar Coordinates 692



8 Systems of Linear Equations and Inequalities 766

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9 Conics, Systems of Nonlinear Equations and Inequalities, and Parametric Equations 906

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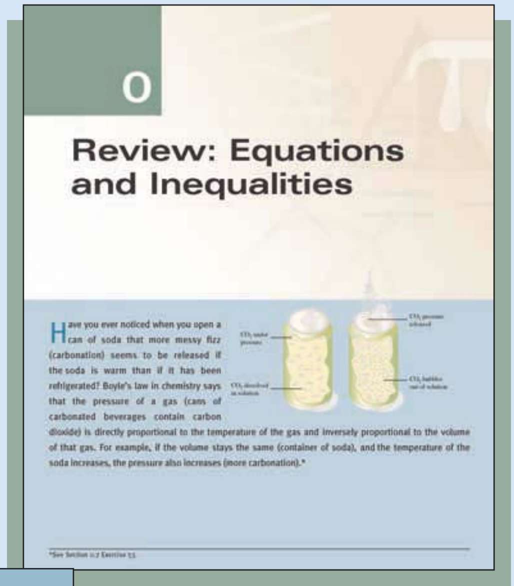
Chapter 11 Limits and the Appendix are available online at www.wiley.com/college/young. For print options including this material, please contact your local Wiley representative.

A Note from the Author to the Student

I wrote this text with careful attention to ways in which to make your learning experience more successful. If you take full advantage of the unique features and elements of this textbook, I believe your experience will be fulfilling and enjoyable. Let's walk through some of the special book features that will help you in your study of algebra and trigonometry.

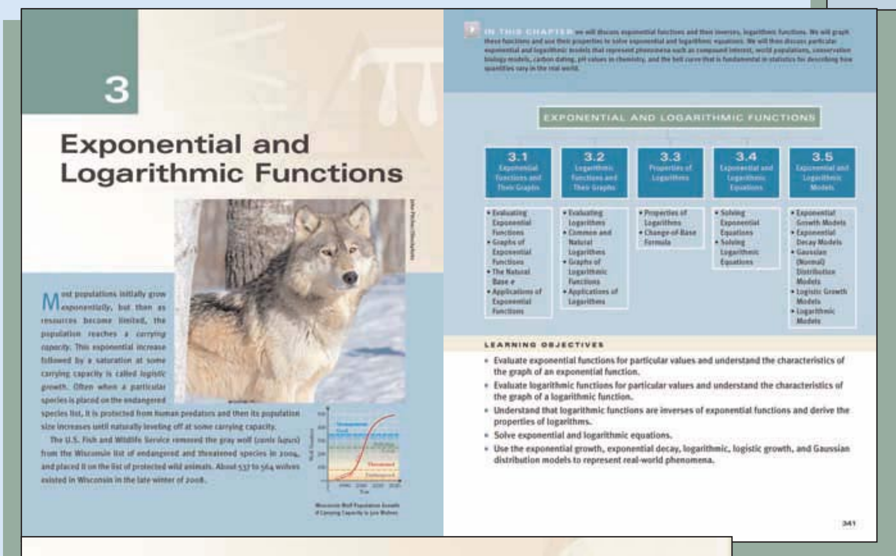


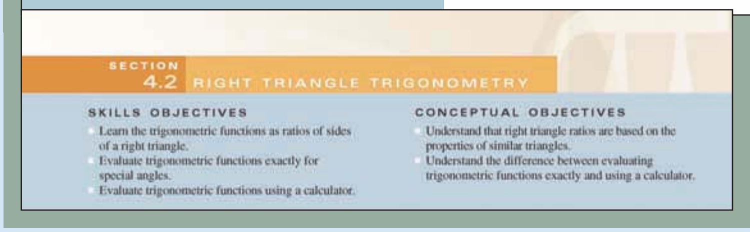
Prerequisites and Review (Chapter 0)

A comprehensive review of prerequisite knowledge (intermediate algebra topics) in Chapter 0 provides a brush up on knowledge and skills necessary for success in the course.

Clear, Concise, and Inviting Writing

Special attention has been made to present an engaging, clear, precise narrative in a layout that is easy to use and designed to reduce any math anxiety you may have.





Chapter Introduction, Flow Chart, Section Headings, and Objectives

An opening vignette, flow chart, list of chapter sections, and chapter learning objectives give you an overview of the chapter.

Skills and Conceptual Objectives

For every section, objectives are further divided by skills *and* concepts so you can see the difference between solving problems and truly understanding concepts.

Examples

Examples pose a specific problem using concepts already presented and then work through the solution. These serve to enhance your understanding of the subject matter.

Your Turn

Immediately following many examples, you are given a similar problem to reinforce and check your understanding. This helps build confidence as you progress in the chapter. These are ideal for in-class activity or for preparing for homework later. Answers are provided in the margin for a quick check of your work.

EXAMPLE 9 Using the Change-of-Base Formula

Use the change-of-base formula to evaluate $\log_4 17$. Round to four decimal places.

Solution:

We will illustrate this in two ways (choosing common and natural logarithms) using a scientific calculator.

Common Logarithms

Use the change-of-base formula with base 10.	$\log_4 17 = \dfrac{\log 17}{\log 4}$
Approximate with a calculator.	≈ 2.043731421
	$\approx \boxed{2.0437}$

Natural Logarithms

Use the change-of-base formula with base e.	$\log_4 17 = \dfrac{\ln 17}{\ln 4}$
Approximate with a calculator.	≈ 2.043731421
	$\approx \boxed{2.0437}$

■ YOUR TURN Use the change-of-base formula to approximate $\log_7 34$. Round to four decimal places.

COMMON MISTAKE

A common mistake is to write the sum of the logs as a log of the sum.

$$\log_b M + \log_b N \neq \log_b(M + N)$$

CORRECT	INCORRECT
Use the power property (7).	
$2\log_b 3 + 4\log_b u = \log_b 3^2 + \log_b u^4$	
Simplify.	
$\log_b 9 + \log_b u^4$	$\neq \log_b(9 + u^4)$ **ERROR**
Use the product property (5).	
$\boxed{= \log_b(9u^4)}$	

Common Mistake/ Correct vs. Incorrect

In addition to standard examples, some problems are worked out both correctly and incorrectly to highlight common errors students make. Counter examples like these are often an effective learning approach for many students.

WORDS	MATH
For a point (x, y) that lies on the unit circle, $x^2 + y^2 = 1$.	$-1 \leq x \leq 1$ and $-1 \leq y \leq 1$
Since $(x, y) = (\cos\theta, \sin\theta)$, the following holds.	$-1 \leq \cos\theta \leq 1$ and $-1 \leq \sin\theta \leq 1$
State the **domain and range of the cosine and sine functions.**	Domain: $(-\infty, \infty)$ Range: $[-1, 1]$
Since $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ and $\csc\theta = \dfrac{1}{\sin\theta}$, the values for θ that make $\sin\theta = 0$	

Parallel Words and Math

This text reverses the common textbook presentation of examples by placing the explanation in words *on the left* and the mathematics in parallel *on the right*. This makes it easier for students to read through examples as the material flows more naturally from left to right and as commonly presented in class.

Study Tips and Caution Notes

These marginal reminders call out important hints or warnings to be aware of related to the topic or problem.

Technology Tips

These marginal notes provide problem solving instructions and visual examples using graphing calculators.

Technology Tip

Use the TI to evaluate the expression for s.

$$s = (6800 \text{ km})(45°)\left(\frac{\pi}{180°}\right)$$

Press 2nd ^ for π. Type 6800 x 45 x 2nd ^ ÷ 180 ENTER.

```
6800*45*π/180
      5340.707511
```

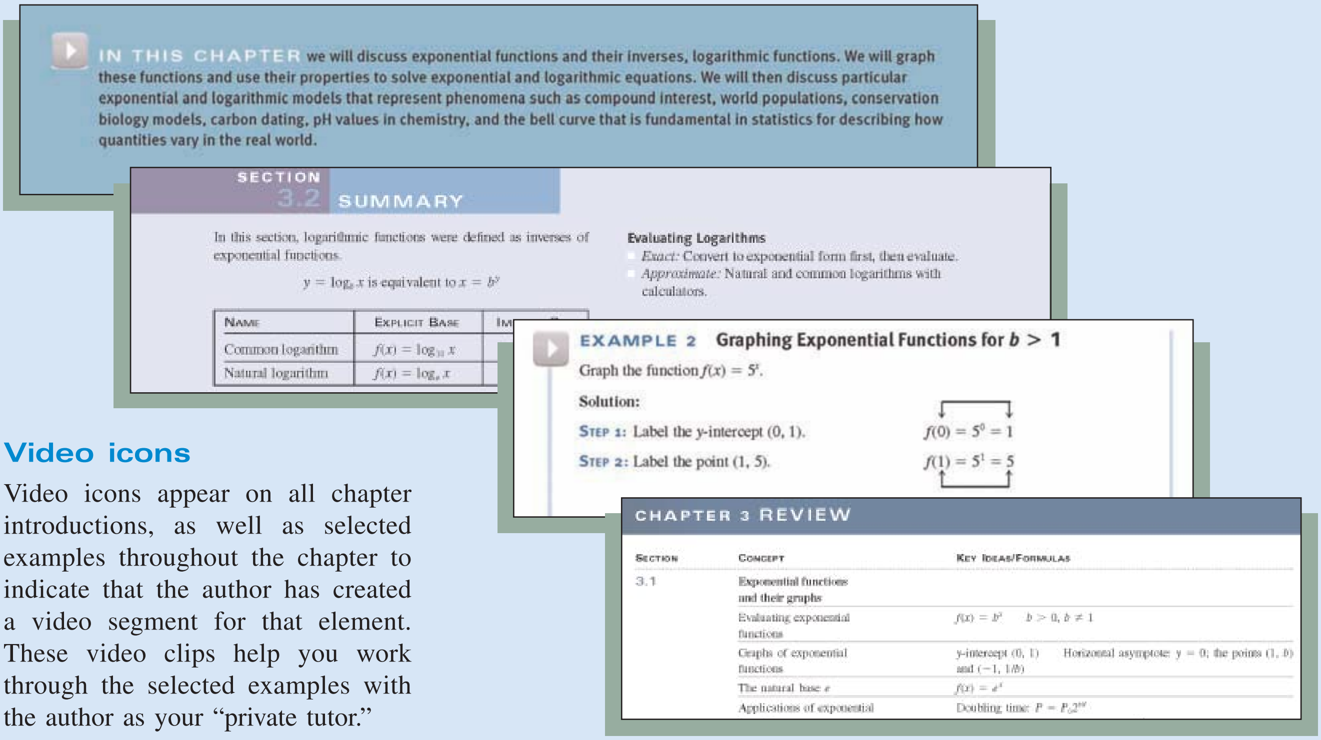
Study Tip

Both the initial side (initial ray) and the terminal side (terminal ray) of an angle are rays.

CAUTION

$$\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$$

$$\log_b M - \log_b N \neq \frac{\log_b M}{\log_b N}$$



IN THIS CHAPTER we will discuss exponential functions and their inverses, logarithmic functions. We will graph these functions and use their properties to solve exponential and logarithmic equations. We will then discuss particular exponential and logarithmic models that represent phenomena such as compound interest, world populations, conservation biology models, carbon dating, pH values in chemistry, and the bell curve that is fundamental in statistics for describing how quantities vary in the real world.

SECTION 3.2 SUMMARY

In this section, logarithmic functions were defined as inverses of exponential functions.

$y = \log_b x$ is equivalent to $x = b^y$

Name	Explicit Base
Common logarithm	$f(x) = \log_{10} x$
Natural logarithm	$f(x) = \log_e x$

Evaluating Logarithms
- *Exact:* Convert to exponential form first, then evaluate.
- *Approximate:* Natural and common logarithms with calculators.

EXAMPLE 2 Graphing Exponential Functions for $b > 1$

Graph the function $f(x) = 5^x$.

Solution:

STEP 1: Label the y-intercept (0, 1). $f(0) = 5^0 = 1$

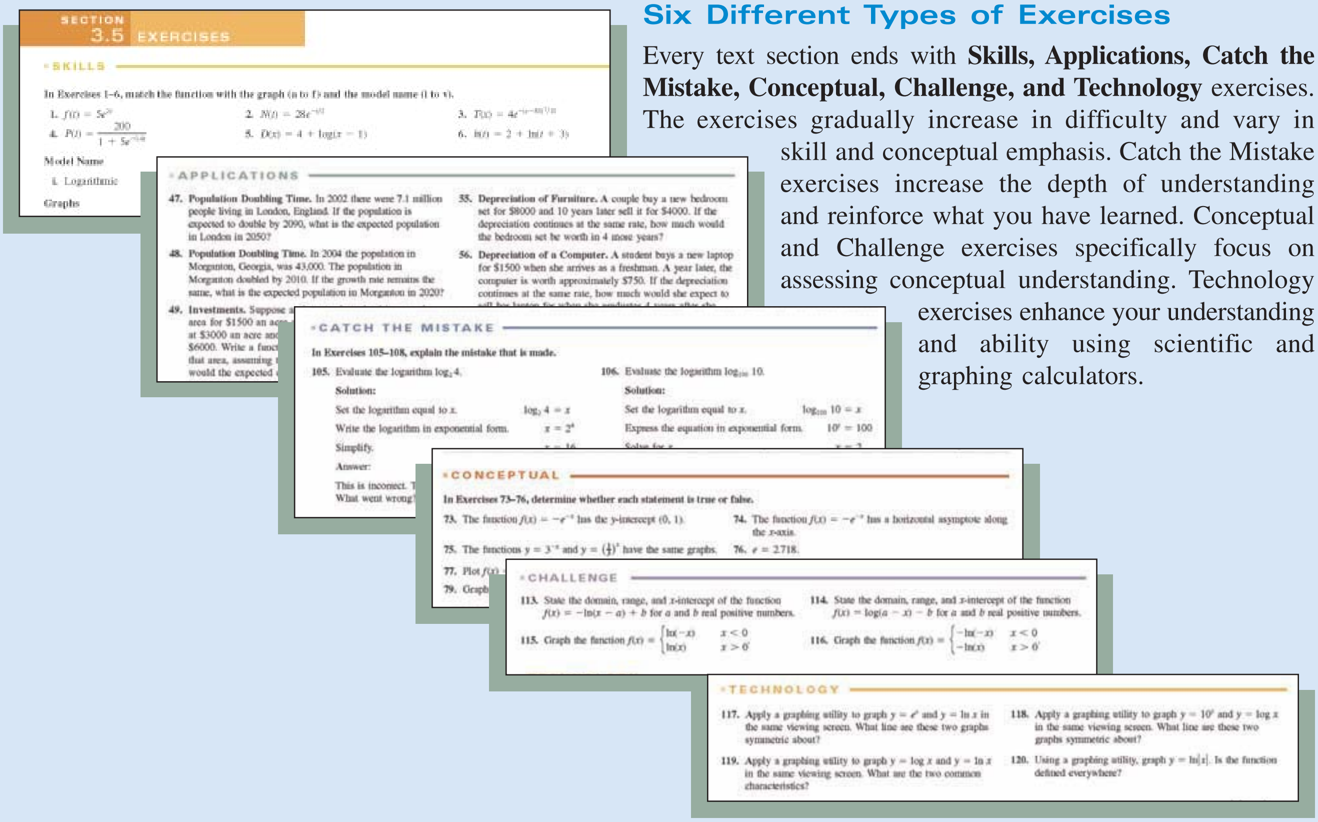
STEP 2: Label the point (1, 5). $f(1) = 5^1 = 5$

CHAPTER 3 REVIEW

Section	Concept	Key Ideas/Formulas
3.1	Exponential functions and their graphs	
	Evaluating exponential functions	$f(x) = b^x$ $b > 0, b \neq 1$
	Graphs of exponential functions	y-intercept (0, 1) Horizontal asymptote: $y = 0$; the points $(1, b)$ and $(-1, 1/b)$
	The natural base e	$f(x) = e^x$
	Applications of exponential	Doubling time: $P = P_0 2^{t/d}$

Video icons

Video icons appear on all chapter introductions, as well as selected examples throughout the chapter to indicate that the author has created a video segment for that element. These video clips help you work through the selected examples with the author as your "private tutor."



SECTION 3.5 EXERCISES

SKILLS

In Exercises 1–6, match the function with the graph (a to f) and the model name (i to v).

APPLICATIONS

47. Population Doubling Time. In 2002 there were 7.1 million people living in London, England. If the population is expected to double by 2090, what is the expected population in London in 2050?

48. Population Doubling Time. In 2004 the population in Morganton, Georgia, was 43,000. The population in Morganton doubled by 2010. If the growth rate remains the same, what is the expected population in Morganton in 2020?

55. Depreciation of Furniture. A couple buy a new bedroom set for \$8000 and 10 years later sell it for \$4000. If the depreciation continues at the same rate, how much would the bedroom set be worth in 4 more years?

56. Depreciation of a Computer. A student buys a new laptop for \$1500 when she arrives as a freshman. A year later, the computer is worth approximately \$750. If the depreciation continues at the same rate, how much would she expect to ...

CATCH THE MISTAKE

In Exercises 105–108, explain the mistake that is made.

105. Evaluate the logarithm $\log_2 4$.

106. Evaluate the logarithm $\log_{100} 10$.

CONCEPTUAL

In Exercises 73–76, determine whether each statement is true or false.

73. The function $f(x) = -e^{-x}$ has the y-intercept (0, 1).

74. The function $f(x) = -e^{-x}$ has a horizontal asymptote along the x-axis.

75. The functions $y = 3^{-x}$ and $y = (\frac{1}{3})^x$ have the same graphs.

76. $e = 2.718$

CHALLENGE

113. State the domain, range, and x-intercept of the function $f(x) = -\ln(x - a) + b$ for a and b real positive numbers.

114. State the domain, range, and x-intercept of the function $f(x) = \log(a - x) - b$ for a and b real positive numbers.

TECHNOLOGY

117. Apply a graphing utility to graph $y = e^x$ and $y = \ln x$ in the same viewing screen. What line are these two graphs symmetric about?

118. Apply a graphing utility to graph $y = 10^x$ and $y = \log x$ in the same viewing screen. What line are these two graphs symmetric about?

119. Apply a graphing utility to graph $y = \log x$ and $y = \ln x$ in the same viewing screen. What are the two common characteristics?

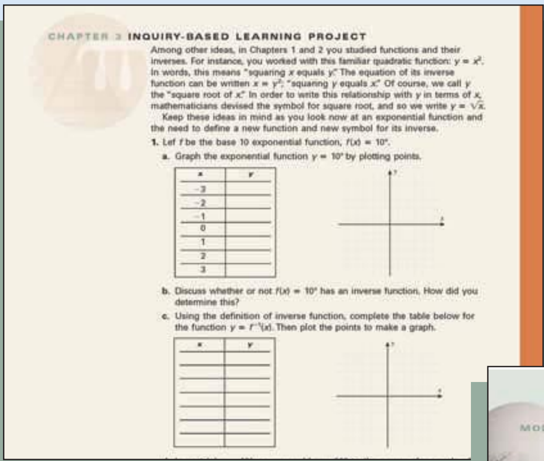
120. Using a graphing utility, graph $y = \ln|x|$. Is the function defined everywhere?

Six Different Types of Exercises

Every text section ends with **Skills, Applications, Catch the Mistake, Conceptual, Challenge, and Technology** exercises. The exercises gradually increase in difficulty and vary in skill and conceptual emphasis. Catch the Mistake exercises increase the depth of understanding and reinforce what you have learned. Conceptual and Challenge exercises specifically focus on assessing conceptual understanding. Technology exercises enhance your understanding and ability using scientific and graphing calculators.

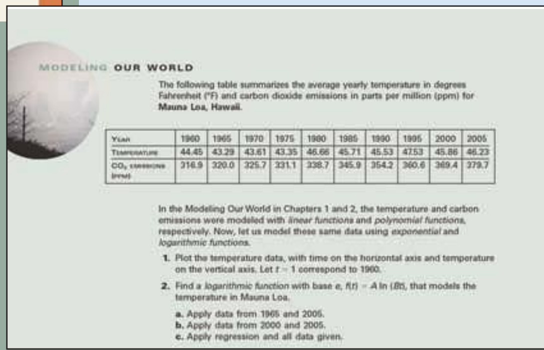
Inquiry-Based Learning Projects

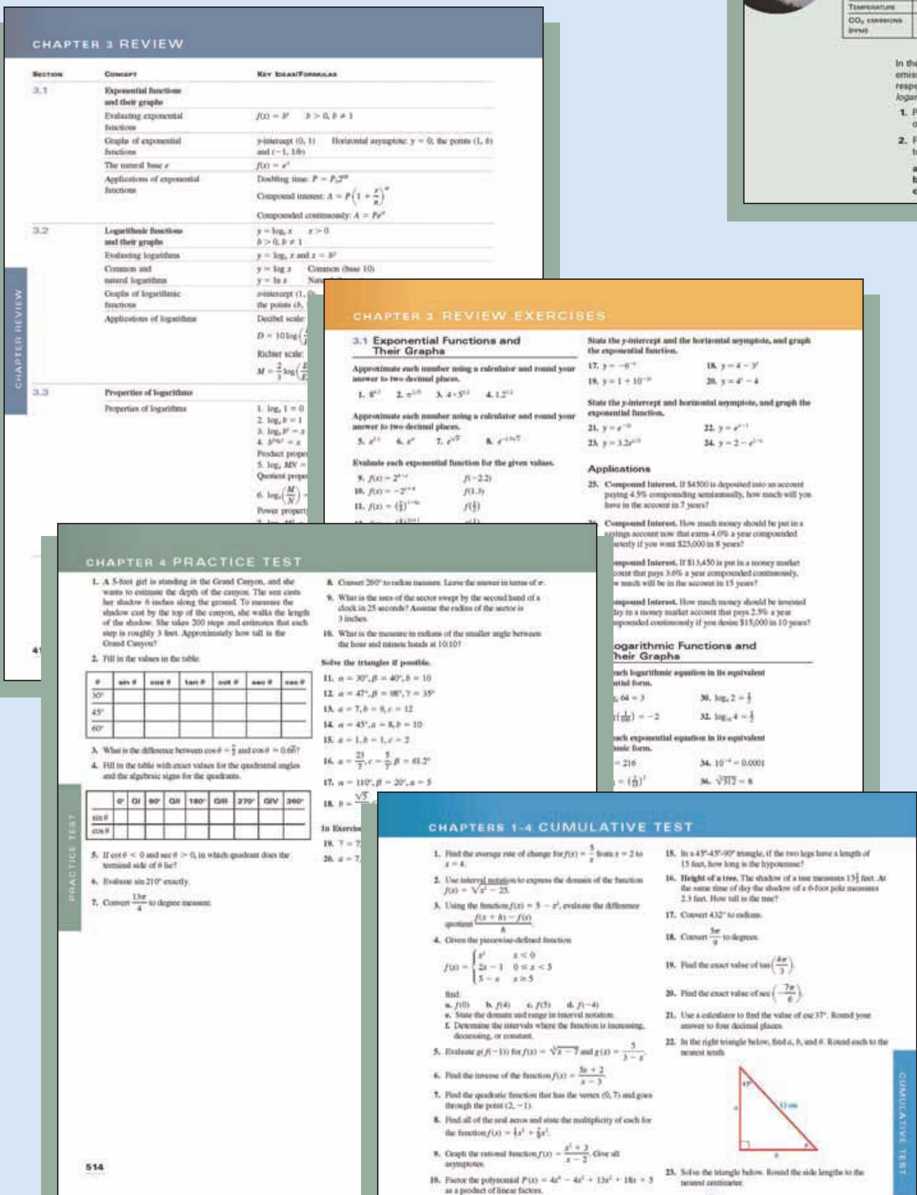
These end of chapter projects enable you to discover mathematical concepts on your own!


CHAPTER 3 INQUIRY-BASED LEARNING PROJECT

Modeling Our World

These unique end-of-chapter exercises provide a fun and interesting way to take what you have learned and model a real world problem. By using climate change as the continuous theme, these exercises can help you to develop more advanced modeling skills with each chapter while seeing how modeling can help you better understand the world around you.


MODELING OUR WORLD


CHAPTER 3 REVIEW

CHAPTER 3 REVIEW EXERCISES

CHAPTER 4 PRACTICE TEST

CHAPTERS 1–4 CUMULATIVE TEST

Chapter Review, Review Exercises, Practice Test, Cumulative Test

At the end of every chapter, a summary review chart organizes the key learning concepts in an easy to use one or two-page layout. This feature includes key ideas and formulas, as well as indicating relevant pages and review exercises so that you can quickly summarize a chapter and study smarter. Review Exercises, arranged by section heading, are provided for extra study and practice. A Practice Test, without section headings, offers even more self-practice before moving on. A new Cumulative Test feature offers study questions based on all previous chapters' content, thus helping you build upon previously learned concepts.

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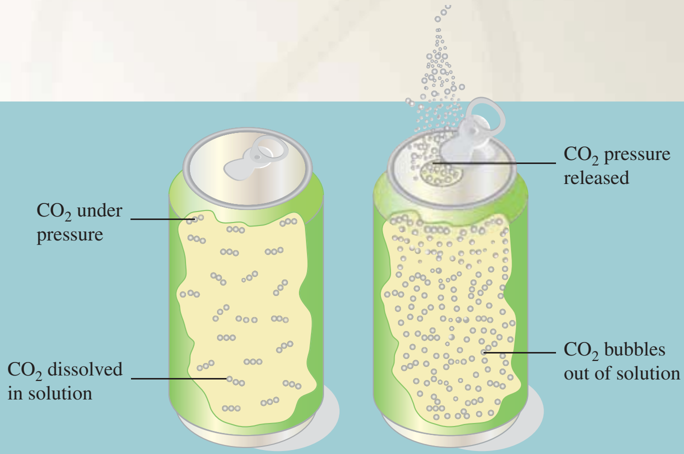
0

Review: Equations and Inequalities

Have you ever noticed when you open a can of soda that more messy fizz (carbonation) seems to be released if the soda is warm than if it has been refrigerated? Boyle's law in chemistry says that the pressure of a gas (cans of carbonated beverages contain carbon dioxide) is directly proportional to the temperature of the gas and inversely proportional to the volume of that gas. For example, if the volume stays the same (container of soda), and the temperature of the soda increases, the pressure also increases (more carbonation).*



*See Section 0.7 Exercise 53.

IN THIS CHAPTER we will review solving equations in one variable. We will start with linear and quadratic equations and then move on to other types of equations. We will review solving linear, polynomial, rational, and absolute value inequalities in one variable. We will discuss how to graph equations in two variables in the Cartesian plane and specifically discuss circles. Lastly, we will use equations to model variation.

REVIEW: EQUATIONS AND INEQUALITIES

0.1 Linear Equations
- Solving Linear Equations in One Variable
- Applications Involving Linear Equations

0.2 Quadratic Equations
- Factoring
- Square Root Method
- Completing the Square
- The Quadratic Formula

0.3 Other Types of Equations
- Rational Equations
- Radical Equations
- Equations Quadratic in Form: *u*-Substitution
- Factorable Equations
- Absolute Value Equations

0.4 Inequalities
- Graphing Inequalities and Interval Notation
- Linear Inequalities
- Polynomial Inequalities
- Rational Inequalities
- Absolute Value Inequalities

0.5 Graphing Equations
- Cartesian Plane
- The Distance and Midpoint Formulas
- Point-Plotting
- Using Intercepts and Symmetry as Graphing Aids
- Circles

0.6 Lines
- Graphing a Line
- Slope
- Equations of Lines
- Parallel and Perpendicular Lines

0.7 Modeling Variation
- Direct Variation
- Inverse Variation
- Joint Variation and Combined Variation

0.8* Linear Regression: Best Fit
- Scatterplots
- Identifying Patterns
- Linear Regression

LEARNING OBJECTIVES

- Solve linear equations in one variable.
- Solve quadratic equations in one variable.
- Solve other types of equations that can be transformed into linear or quadratic equations.
- Solve inequalities in one variable.
- Graph equations in two variables in the Cartesian plane.
- Find the equation of a line.
- Use equations to model variation.
- Find the line of best fit for a given data set.*

*Optional Technology Required Section.

SECTION 0.1 LINEAR EQUATIONS

SKILLS OBJECTIVES

- Solve linear equations in one variable.
- Solve application problems involving linear equations.

CONCEPTUAL OBJECTIVE

- Understand the mathematical modeling process.

Solving Linear Equations in One Variable

An **algebraic expression** (see Appendix) consists of one or more terms that are combined through basic operations such as addition, subtraction, multiplication, or division; for example,

$$3x + 2 \qquad 5 - 2y \qquad x + y$$

An **equation** is a statement that says two expressions are equal. For example, the following are all equations in one variable, x:

$$x + 7 = 11 \qquad x^2 = 9 \qquad 7 - 3x = 2 - 3x \qquad 4x + 7 = x + 2 + 3x + 5$$

To **solve** an equation in one variable means to find all the values of that variable that make the equation true. These values are called **solutions**, or **roots**, of the equation. The first of these statements shown above, $x + 7 = 11$, is true when $x = 4$ and false for any other values of x. We say that $x = 4$ is the solution to the equation. Sometimes an equation can have more than one solution, as in $x^2 = 9$. In this case, there are actually two values of x that make this equation true, $x = -3$ and $x = 3$. We say the **solution set** of this equation is $\{-3, 3\}$. In the third equation, $7 - 3x = 2 - 3x$, no values of x make the statement true. Therefore, we say this equation has **no solution**. And the fourth equation, $4x + 7 = x + 2 + 3x + 5$, is true for any values of x. An equation that is true for any value of the variable x is called an **identity**. In this case, we say the solution set is the **set of all real numbers**.

Two equations that have the same solution set are called **equivalent equations**. For example,

$$3x + 7 = 13 \qquad 3x = 6 \qquad x = 2$$

are all equivalent equations because each of them has the solution set $\{2\}$. Note that $x^2 = 4$ is not equivalent to these three equations because it has the solution set $\{-2, 2\}$.

When solving equations, it helps to find a simpler equivalent equation in which the variable is isolated (alone). The following table summarizes the procedures for generating equivalent equations.

Generating Equivalent Equations

Original Equation	Description	Equivalent Equation
$3(x - 6) = 6x - x$	■ Eliminate the parentheses. ■ Combine like terms on one or both sides of the equation.	$3x - 18 = 5x$
$7x + 8 = 29$	Add (or subtract) the same quantity to (from) *both* sides of the equation. $7x + 8 - \mathbf{8} = 29 - \mathbf{8}$	$7x = 21$
$5x = 15$	Multiply (or divide) both sides of the equation by the same nonzero quantity: $\frac{5x}{5} = \frac{15}{5}$.	$x = 3$
$-7 = x$	Interchange the two sides of the equation.	$x = -7$

You probably already know how to solve simple linear equations. Solving a linear equation in one variable is done by finding an equivalent equation. In generating an equivalent equation, remember that whatever operation is performed on one side of an equation must also be performed on the other side of the equation.

EXAMPLE 1 **Solving a Linear Equation**

Solve the equation $3x + 4 = 16$.

Solution:

Subtract 4 from both sides of the equation.

$$\begin{array}{r} 3x + 4 = 16 \\ -4 \quad -4 \\ \hline 3x \quad = 12 \end{array}$$

Divide both sides by 3.

$$\frac{3x}{3} = \frac{12}{3}$$

The solution is $x = 4$.

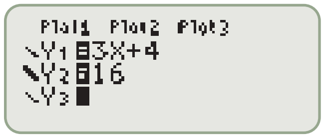
$$\boxed{x = 4}$$

The solution set is $\{4\}$.

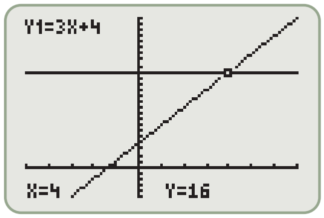
■ **YOUR TURN** Solve the equation $2x + 3 = 9$.

Technology Tip

Use a graphing utility to display graphs of $y_1 = 3x + 4$ and $y_2 = 16$.



The x-coordinate of the point of intersection is the solution to the equation $3x + 4 = 16$.



■ **Answer:** The solution is $x = 3$. The solution set is $\{3\}$.

Example 1 illustrates solving linear equations in one variable. What is a linear equation in one variable?

DEFINITION **Linear Equation**

A **linear equation in one variable**, x, can be written in the form

$$ax + b = 0$$

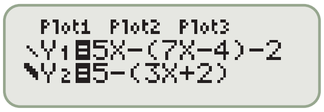
where a and b are real numbers and $a \neq 0$.

What makes this equation linear is that x is raised to the first power. We can also classify a linear equation as a **first-degree** equation.

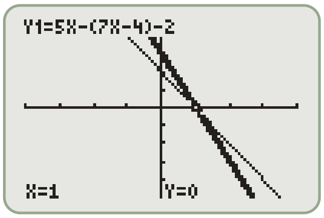
Equation	**Degree**	**General Name**
$x - 7 = 0$	First	Linear
$x^2 - 6x - 9 = 0$	Second	Quadratic
$x^3 + 3x^2 - 8 = 0$	Third	Cubic

Technology Tip

Use a graphing utility to display graphs of $y_1 = 5x - (7x - 4) - 2$ and $y_2 = 5 - (3x + 2)$.



The x-coordinate of the point of intersection is the solution to the equation $5x - (7x - 4) - 2 = 5 - (3x + 2)$.



EXAMPLE 2 Solving a Linear Equation

Solve the equation $5x - (7x - 4) - 2 = 5 - (3x + 2)$.

Solution:

Eliminate the parentheses. $\quad 5x - (7x - 4) - 2 = 5 - (3x + 2)$

Don't forget to distribute the negative sign through *both* terms inside the parentheses. $\quad 5x - 7x + 4 - 2 = 5 - 3x - 2$

Combine like terms on each side. $\quad -2x + 2 = 3 - 3x$

Add $3x$ to both sides. $\quad +3x \qquad + 3x$

$x + 2 = 3$

Subtract 2 from both sides. $\quad -2 \qquad -2$

$x = 1$

Check to verify that $x = 1$ is a solution to the original equation.

$$5 \cdot 1 - (7 \cdot 1 - 4) - 2 = 5 - (3 \cdot 1 + 2)$$
$$5 - (7 - 4) - 2 = 5 - (3 + 2)$$
$$5 - (3) - 2 = 5 - (5)$$
$$0 = 0$$

Since the solution $x = 1$ makes the equation true, the solution set is $\{1\}$.

■ **YOUR TURN** Solve the equation $4(x - 1) - 2 = x - 3(x - 2)$.

■ **Answer:** The solution is $x = 2$. The solution set is $\{2\}$.

Study Tip

Prime Factors

$2 = 2$

$6 = 2 \cdot 3$

$5 = \quad \cdot 5$

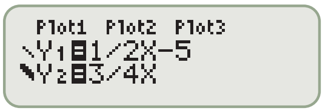
$\text{LCD} = 2 \cdot 3 \cdot 5 = 30$

To solve a linear equation involving fractions, find the least common denominator (LCD) of all terms and multiply both sides of the equation by the LCD. We will first review how to find the LCD.

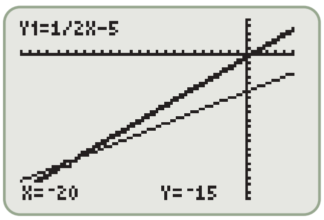
To add the fractions $\frac{1}{2} + \frac{1}{6} + \frac{2}{5}$, we must first find a common denominator. Some people are taught to find the lowest number that 2, 6, and 5 all divide evenly into. Others prefer a more systematic approach in terms of prime factors.

Technology Tip

Use a graphing utility to display graphs of $y_1 = \frac{1}{2}p - 5$ and $y_2 = \frac{3}{4}p$.



The x-coordinate of the point of intersection is the solution.



EXAMPLE 3 Solving a Linear Equation Involving Fractions

Solve the equation $\frac{1}{2}p - 5 = \frac{3}{4}p$.

Solution:

Write the equation. $\quad \dfrac{1}{2}p - 5 = \dfrac{3}{4}p$

Multiply each term in the equation by the LCD, 4. $\quad (4)\dfrac{1}{2}p - (4)5 = (4)\dfrac{3}{4}p$

The result is a linear equation with no fractions. $\quad 2p - 20 = 3p$

Subtract $2p$ from both sides. $\quad -2p \qquad -2p$

$-20 = p$

$\boxed{p = -20}$

Since $p = -20$ satisfies the original equation, the solution set is $\{-20\}$.

■ **YOUR TURN** Solve the equation $\frac{1}{4}m = \frac{1}{12}m - 3$.

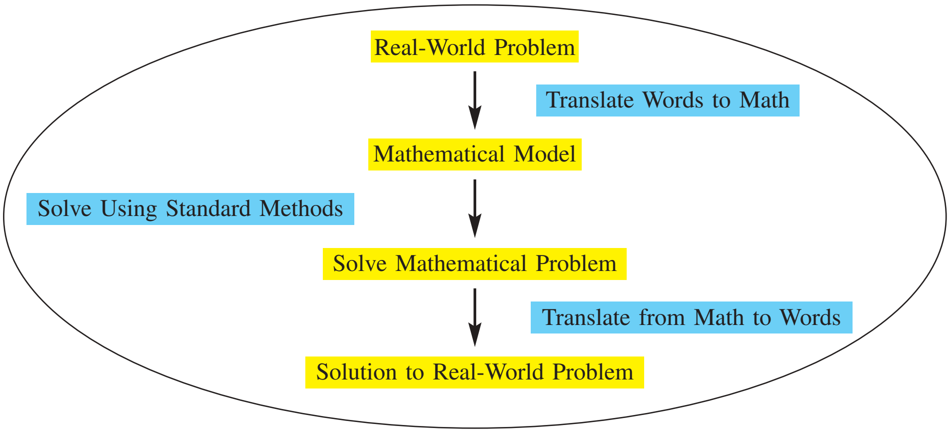
■ **Answer:** The solution is $m = -18$. The solution set is $\{-18\}$.

Solving a Linear Equation in One Variable

Step	Description	Example
1	Simplify the algebraic expressions on both sides of the equation.	$-3(x-2)+5=7(x-4)-1$ $-3x+6+5=7x-28-1$ $-3x+11=7x-29$
2	Gather all variable terms on one side of the equation and all constant terms on the other side.	$-3x+11=7x-29$ $+3x \quad\quad +3x$ $11=10x-29$ $+29 \quad\quad +29$ $40=10x$
3	Isolate the variable.	$10x=40$ $\boxed{x=4}$

Applications Involving Linear Equations

We now use linear equations to solve problems that occur in our day-to-day lives. You typically will read the problem in words, develop a mathematical model (equation) for the problem, solve the equation, and write the answer in words.



You will have to come up with a unique formula to solve each kind of word problem, but there is a universal *procedure* for approaching all word problems.

Procedure for Solving Word Problems

Step 1: **Identify the question.** Read the problem *one* time and note what you are asked to find.

Step 2: **Make notes.** Read until you can note something (an amount, a picture, anything). Continue reading and making notes until you have read the problem a second* time.

Step 3: **Assign a variable to whatever is being asked for.** If there are two choices, then let it be the smaller of the two.

Step 4: **Set up an equation.** Assign a variable to represent what you are asked to find.

Step 5: **Solve the equation.**

Step 6: **Check the solution.** Substitute the solution for the variable in the equation, and also run the solution past the "common sense department" using estimation.

*Step 2 often requires multiple readings of the problem.

EXAMPLE 4 How Long Was the Trip?

During a camping trip in North Bay, Ontario, a couple went one-third of the way by boat, 10 miles by foot, and one-sixth of the way by horse. How long was the trip?

Solution:

STEP 1 Identify the question.
How many miles was the trip?

STEP 2 Make notes.

Read	Write
... one-third of the way by boat	BOAT: $\frac{1}{3}$ of the trip
... 10 miles by foot	FOOT: 10 miles
... one-sixth of the way by horse	HORSE: $\frac{1}{6}$ of the trip

STEP 3 Assign a variable.
Distance of total trip in miles $= x$

STEP 4 Set up an equation.
The total distance of the trip is the sum of all the distances by boat, foot, and horse.

Distance by boat + Distance by foot + Distance by horse = Total distance of trip

Distance by boat $= \frac{1}{3}x$

Distance by foot $= 10$ miles

Distance by horse $= \frac{1}{6}x$

$$\underbrace{\frac{1}{3}x}_{\text{boat}} + \underbrace{10}_{\text{foot}} + \underbrace{\frac{1}{6}x}_{\text{horse}} = \underbrace{x}_{\text{total}}$$

STEP 5 Solve the equation.

$$\frac{1}{3}x + 10 + \frac{1}{6}x = x$$

Multiply by the LCD, 6. $\quad 2x + 60 + x = 6x$

Collect x terms on the right. $\quad 60 = 3x$

Divide by 3. $\quad 20 = x$

The trip was 20 miles. $\quad x = 20$

STEP 6 Check the solution.
Estimate: The boating distance, $\frac{1}{3}$ of 20 miles, is approximately 7 miles; the riding distance on horse, $\frac{1}{6}$ of 20 miles, is approximately 3 miles. Adding these two distances to the 10 miles by foot gives a trip distance of 20 miles.

■ **YOUR TURN** A family arrives at the Walt Disney World parking lot. To get from their car in the parking lot to the gate at the Magic Kingdom, they walk $\frac{1}{4}$ mile, take a tram for $\frac{1}{3}$ of their total distance, and take a monorail for $\frac{1}{2}$ of their total distance. How far is it from their car to the gate of the Magic Kingdom?

■ **Answer:** The distance from their car to the gate is 1.5 miles.

Geometry Problems

Some problems require geometric formulas in order to be solved.

EXAMPLE 5 Geometry

A rectangle 24 meters long has the same area as a square with 12-meter sides. What are the dimensions of the rectangle?

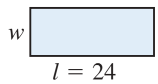
Solution:

STEP 1 Identify the question.
What are the dimensions (length and width) of the rectangle?

STEP 2 Make notes.

Read — A rectangle 24 meters long

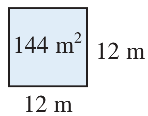
Write/Draw



area of rectangle $= 1 \cdot w = 24w$

Read — A square with 12-meter sides

Write/Draw



area of square $= 12 \cdot 12 = 144$

STEP 3 Assign a variable. Let w = width of the rectangle.

STEP 4 Set up an equation.

The area of the rectangle is equal to the area of the square. rectangle area = square area

Substitute in known quantities. $24w = 144$

STEP 5 Solve the equation.

Divide by 24. $w = \frac{144}{24} = 6$

The rectangle is 24 meters long and 6 meters wide.

STEP 6 Check the solution.
A 24 meter by 6 meter rectangle has an area of 144 square meters.

YOUR TURN A rectangle 3 inches wide has the same area as a square with 9-inch sides. What are the dimensions of the rectangle?

Answer: The rectangle is 27 in. long and 3 in. wide.

Interest Problems

In our personal or business financial planning, a particular concern we have is interest. **Interest** is money paid for the use of money; it is the cost of borrowing money. The total amount borrowed is called the **principal**. The principal can be the price of our new car; we pay the bank interest for loaning us money to buy the car. The principal can also be the amount we keep in a CD or money market account; the bank uses this money and pays us